98th Annual AAAS-PD Meeting
19-23 June 2017
Waimea, Big Island, Hawai`i

REVISED SCHWARZSCHILD SOLUTION TO ACCOMMODATE SPACE EXPANSION

By* Thomas E. Chamberlain, Ph.D.

ABSTRACT

Deep MOND correlations show far-field gravitation around any spiral galaxy declining inversely with radius, contrary to inverse-radius squared of Newtonian gravitation and general relativity. The required a~1/r theory is obtained by first introducing a fundamentally distinct alternative to relativity physics at the core of special and (therefore) general relativity—specifically, Einstein's c=constant is replaced by infinite photon-speed radially inward and $c/2$ radially outward. By itself this step adds terms (e.g., to Maxwell's equations) without changing predictions. But within Hubble space-expansion, the resulting $\mathrm{d}\Delta t'/\mathrm{d}t = -V_H/c = -r_H cH/c^2$ combined with the Schwarzschild (linearized) time-dilation $\mathrm{d}\Delta t'/\mathrm{d}t = -GM/r_S c^2$ gives a new, *inductively-formulated* time-dilation $\mathrm{d}\Delta t'/\mathrm{d}t = -([GM/r_S][r_H cH])^{1/2}/c^2$ which is spatially uniform in uniform *fundamental-observer* time. Despite this uniform time-dilation in any given epoch, for non-accelerating expansion "$-r_H cH/c^2$" becomes the invariant "$-r_H c(r_0 H_0/r_H)/c^2$" and radial differentiation of "$-([GM/r_S][r_0 cH_0])^{1/2}/c^2$" yields non-uniform time-dilation; gravitational acceleration $a = \mathrm{d}([GM/r_S][r_0 cH_0])^{1/2}/\mathrm{d}r = -½(GMcH_0)^{1/2}/r$ necessary to explain spiral galaxy rotation flattening analytically follows. Applying the corresponding metric within the Schwarzschild solution gives x10^{-3} to x10^{-4} additional gravitational acceleration across the inner planets of the Solar System, which can be reduced by Solar and planetary mass adjustments. Newtonian gravitational acceleration decreases to equal the new gravitational component at ~7000 AU, in accord with recent wide-binary star rotation data, and a~1/r gradually dominates at greater distances where dark matter could be evoked to explain the discrepancy. However, dark matter yields the same a~1/r as Deep MOND around spiral galaxies, and the present deeper theory of gravity, also with a~1/r, may be acceptable due its greater scientific promise—e.g., a new theoretical relationship between time and matter.

* Independent Researcher. Los Angeles, CA. Rev 1; November 29, 2017.
thomas.e.chamberlain@gmail.com

1.0 Introduction

In their paper "The Influence of the Expansion of Space on the Gravitational Fields Surrounding the Individual Stars" Einstein and Straus (1945) concluded:

> "…the expansion of space has no influence on the structure of the field surrounding an individual star, that it is a static field—if only for an exactly delimited neighborhood."

Because anisotropic light-speed was not measurable (and remains unmeasurable to the present day) strictly isotropic light-speed was confidently retained and the influence of space expansion accordingly dismissed.

In recent years, however, theoretical studies of anisotropic light-speed at the foundation of relativity physics—e.g., Rizzi et al. (2008) and Chamberlain (2015)[1]—have revealed a conceptual and mathematical richness that invites exploration, albeit with no immediate empirical support. Furthermore, for several decades astrophysical measurements of spiral-galaxy rotation flattening have exhibited a progressive breakdown of Newtonian gravity and general relativity below a_0=1.2E-13 km/s^2 (universal acceleration "constant";[2] Milgrom (1983)). Additionally, a close—within an order—relationship exists between a_0 and cH_0 (i.e., 1.2E-13 km/s^2 versus 6.3E-13 km/s^2; later in the paper $a_0=cH_0/4$ is established). These considerations suggest that investigation of anisotropic light speed within the Hubble expansion may reveal a corresponding influence on the structure of gravitational fields.

The manner in which anisotropic light-speed might help explain asymptotic flattening of spiral galaxy (and binary-star) rotation is explored in the present work. Investigation began immediately after presentation of "Fundamentally Anisotropic Light-velocity at the Foundation of Classical Physics" (title changed) during the June 2015 annual conference of AAAS-PD in San Francisco, wherein Einstein's same-motion acceleration/synchrony (1907) was employed to exhibit—among several developments—the multi-state character of classical reality (due to standard re-synchronization after an interval of synchronous same-motion acceleration). Shortly thereafter, the development of a same-motion model of the Hubble expansion yielded a near vanishingly-small acceleration cH_0 that matched Milgrom's a_0 within an order of magnitude (as mentioned above).

Because the model gives inwardly infinite photon velocity (i.e., opposite to same-motion velocity) *across uniform time* it cannot be entirely meaningful—i.e., it would allow near-instantaneous (i.e., back and forth) communication within any given epoch. This ambiguity was resolved by (inductively) postulating a negative time-gradient forward from the linearly accelerated observer against which inward photons progressed. The resulting *expansion-imposed* time dilation, $d\Delta t'/dt = -rH/c$, was then (again, inductively) combined with the Schwarzschild time dilation, $d\Delta t'/dt = -GM/r_S c^2$, to arrive at a new time dilation which is radially invariant in any given epoch, $d\Delta t'/dt = -([GM/r_S][r_H cH])^{1/2}/c^2 = -(GMcH)^{1/2}/c^2$, surrounding any given mass concentration (e.g., galaxy, star, planet, etc.).

How $1/r$ dependence of far-field gravitation emerged from the new time-dilation was the next question. Stipulating non-accelerating space expansion was the key thereby yielding $a = -\frac{1}{2}(GMcH_0)^{1/2}/r$ in our present epoch, which is the relativistic formulation of Milgrom's non-relativistic *Deep MOND*. This particular development—i.e., the emergence of $1/r$ gravitation from radial differentiation of $(GMcH)^{1/2}$—exhibited a correspondence to the emergence of Newtonian $1/r^2$ gravitation in the Schwarzschild solution (i.e., after the non-relativistic limitation). The corresponding metric within the

[1] The writer agrees with Rizzi et al. in their tribute to Franco Selleri for his important contributions, in which they offer … "[a] new independent light is thrown on SRT [requiring] a suitable reformulation of the theory".
[2] "Constant" is in quotations because Milgrom's universal acceleration a_0 declines with time—in accordance with the decline of the Hubble parameter (i.e., $a_0 \sim cH(t)$), as will be addressed later in the paper. Hereafter we will substitute "parameter" for "constant".

Schwarzschild geometry then led to a revised Schwarzschild solution that exhibits an additional $1/r$ "sub-field" gravitation.

In the following discussion we first develop sub-field time dilation and the corresponding gravitational acceleration (Section 2.0). Next, with the recognition that space-time curvature attends the new mode of gravitational acceleration, the sub-field metric is defined, thereby allowing a revised Schwarzschild solution (Section 3.0). Comparison of present theory with standard theory is given in Section 4.0, wherein predicted gravity for the Mercury orbit agrees with the original Schwarzschild solution to within 0.01%. In Section 5.0, however, 1/r sub-field gravity and $1/r^2$ Newtonian/general-relativity gravity around the Sun are approximately equal at ~7000 AU, in accord with recent binary-star rotation-flattening measurements. At much greater distances measured from the center of any given spiral galaxy, predicted outer velocities are in close agreement with Milgrom's Deep MOND and the baryonic Tully-Fisher relation. Concluding remarks are provided in Section 6.0.

2.0 Sub-Field Gravitation Due To Hubble Space Expansion

2.1 Same-Motion Synchrony

> "*… any two clocks of [accelerated system] Σ are synchronous with respect to [nonaccelerated reference system] S at the time t = 0, and undergo the same motion, they remain continuously synchronous with respect to S.*
> *On the other hand, we must not consider the [same-motion] local time σ as simply the "time" of Σ, because, in fact, two [clocks] at two different points of Σ are not [synchronous] in the sense of [special relativity] when their local times σ are equal to each other*." Quotations in reversed order. (Albert Einstein, Principle of Relativity and Gravitation, 1907, p. 900)

Consider the Schwarzschild solution of the Einstein field equations. Suppose photons passing radially inward through any infinitesimal volume move at infinite speed inward and half-c outward (thereby satisfying the *round-trip axiom* derived from the Michelson-Morley experiment) instead of the orthodox c=constant. Schwarzschild's solution of the Einstein equations would remain unchanged. But this is not true, however, in expanding space—i.e., within the Hubble expansion: a new relativistic gravitation with inverse-radius dependence emerges, one that dominates Newtonian gravitation beyond ~7000 AU in the Solar System and beyond the Solar System in the Milky Way Galaxy.

Milgrom's Deep MOND—i.e., $a=(GMa_0)^{1/2}/r$, with universal acceleration parameter a_0=1.2E-13 km/s^2—stands as an important clue for determining why general relativity fails to predict star velocity flattening in wide-binaries and galaxies. In seeking the relativistic explanation for empirical, non-relativistic Deep MOND, Einstein's same-motion acceleration/synchrony enabled a "Hubble-like" space-expansion model that yielded cH_0 close to a_0 (~ same order) and inwardly-infinite photon speed. While the model is unrealistic—as a particular, observers would see distant events in present versus earlier times—it formed the basis for inductive postulates leading to the deeper theory.

2.1.1 Hubble Expansion Modeled by Same-Motion Acceleration Gives a_0 and cH_0 of the Same Order ($cH_0/4=a_0$ is later determined.)

The progress of science may be understood as inductive advances based on empirical discoveries followed by deductive "working out" the consequences. In this understanding, Deep MOND—which has successfully correlated the outer (flat) rotations of spiral galaxies measured over the past several decades—comprises a modern empirical discovery, and the advances to follow present some of the follow-on inductive and deductive developments.

Einstein's same-motion synchrony is the foundation for the inductive advances. For this relativistic principle $(1)_1$ of Chamberlain (2015) becomes:

$$\Delta x' = \Delta x/\gamma .$$

Taking the time-derivative yields

$$\begin{aligned} \mathrm{d}\Delta x'/\mathrm{d}t' &= \mathrm{d}(\Delta x/\gamma)/\mathrm{d}t' \\ &= \Delta x\ \mathrm{d}(1/\gamma)/\mathrm{d}t \bullet \mathrm{d}t/\mathrm{d}t' \\ &= \gamma\,\Delta x'\ (\beta\,\mathrm{d}\beta\,/\mathrm{d}t)/\gamma^3/\gamma \\ &= \beta\Delta x' a'/c \end{aligned} \qquad (1)$$

where a' is acceleration along the x'-axis. This expression means that when two at-rest test particles initially Δx apart along the x-axis are same-motion accelerated along the axis their relative speed in the moving frame increases in proportion to the speed ratio $\beta = v/c$. Note that when same-motion acceleration instantly stops, the rate of separation becomes instantly zero.

We now recognize a significant *quantitative* correspondence—that between separating-yet-synchronous clocks experiencing same motion acceleration and separating-yet-synchronous clocks experiencing Hubble space-expansion (i.e., synchronous in accordance with the Cosmological Principle). In demonstrating this correspondence, we use (1) to calculate the same motion acceleration that duplicates Hubble expansion, finding that it equals Milgrom's universal acceleration parameter, a_0=1.2E-13 km/s^2, to within an order of magnitude. Specifically, from (1) we may write $\mathrm{d}\Delta x'/\mathrm{d}t'=\beta\Delta x' a'/c=rH$ resulting in $a'=cH/\beta$. For β approaching unity—which gives reversed (i.e., opposite to acceleration direction) light-velocity approaching infinity in the moving system—the relation yields a'=3E5 km/s•2.1E-18 km/s/km=6.3E-13 km/s^2 which is approximately the same order as a_0 noted above.

Notice, however, that the "quantitative correspondence" between Hubble space-expansion and same-motion acceleration for $\beta\to 1$ yields a profound departure from our astrophysical observations (and orthodox theory), because were photons entering our present-day telescopes indeed coming in at infinite speed in accord with the above, we would observe distant galaxies as they are today, not millions (and billions) of years ago. This means that the above can only be a clue and not a correct conclusion. We accordingly arrive at the first inductive advance, developed immediately below, that photon transmission—from across the room or across the cosmos—is not only instantaneous but originates in an earlier epoch (thereby leading to an influence of Hubble space-expansion that has been previously overlooked (Einstein and Straus, 1945)).

2.1.2 Inwardly Infinite Light-Velocity

While isotropic, c=constant light-speed has been the standard for over 100 years we are now recognizing that light speed is fundamentally *anisotropic*, where a photon may propagate at faster than nominal light speed in one direction and then backwards at a slower speed while complying with the Michelson-Morley "round trip axiom"—e.g., 100c forward and 0.513c backward. This non-measurable condition has been properly ignored over the decades, but telescopic measurements of spiral-galaxy rotation flattening (including our Galaxy) point to a possible breakdown of gravity theory, the resolution of which could deepen our understanding of space-time with the prospect of follow-on advances. For this reason it is appropriate to explore how anisotropic light-speed might help resolve the "missing mass" problem of astrophysics.

Einstein's same-motion acceleration/synchrony provides guidance in this task. We recall from the previous section that same-motion acceleration to a speed v~c yields near-infinite light-speed in the reverse direction while (approximately) matching Hubble expansion in the forward direction. This was taken as a clue—when it was first realized shortly after the San Francisco AAAS-PD Conference—and one-way infinite light-speed (inward) was adopted as an overarching principle in the subsequent investigation and development.

Qualification of this principle became immediately necessary in order to comply with decades, and indeed centuries, of measurements showing earlier time with increasing distance. Hence, light emitted from distant events continues to arrive instantly albeit passing through positive time-gradients giving the illusion of elapsed time. This is of course a major departure from our c=constant understanding of relativity. But, as noted, the idea is in conflict only with standard *mathematical* physics and not *empirical* physics. And it suggests a beneficial correspondence with similarly counterintuitive quantum mechanics.

2.2 Time-Dilation and Gravitation within the Hubble Flow

Revision of general relativity to accommodate sub-field gravitation around the individual mass-concentration (e.g., star, galaxy, or galactic cluster) begins with mathematical formulation of the attending time dilation. In this basically inductive process two separate/unique time-dilations are joined to yield an overarching time dilation, uniform throughout the sub-field of the massive entity.

Considered first is the time-dilation—$\mathrm{d}\Delta t'/\mathrm{d}t = -r_H H/c$—attending a "Hubble-receding" fundamental observer at distance r_H as determined by another fundamental observer at r=0 who receives the (inwardly-infinite) photons. This development is immediately followed by the linearized time-dilation— $\mathrm{d}\Delta t'/\mathrm{d}t = -GM/r_S c^2$ —derived from the Schwarzschild solution of Einstein's field equations. The two time-dilations are then (again, inductively) joined to give the sub-field time-dilation in any given epoch— $\mathrm{d}\Delta t'/\mathrm{d}t = -(GMcH)^{1/2}/c^2$.

In developing the time-dilation attending a receding fundamental observer exclusive of the Schwarzschild gravitational time dilation we begin with the Lorentz transformation in Cartesian coordinates using familiar notation:

$$\begin{aligned} x' &= (x - \beta ct)/\gamma \\ t' &= (t - \beta/c\, x)/\gamma \\ y' &= y \\ z' &= z \end{aligned} \tag{2}$$

Substituting x from $(2)_1$ allows $(2)_2$ to be written

$$t' = \gamma t - (\beta/c)\, x'$$

or, in differential form with β=0,

$$\mathrm{d}\Delta t'/\mathrm{d}t = -\tfrac{1}{2}\,(v/c)^2\ . \tag{3}$$

Equation (3) shows the relationship between time dilation and speed of a test particle in a given inertial system. If we now consider a clock moving radially outward at speed $V_H = r_H H$ within the Hubble flow, its time dilation virtually seen (through a sufficiently powerful telescope) by a fundamental observer at r=0 (for inwardly infinite photon speed) is

$$\mathrm{d}\Delta t'/\mathrm{d}t = -\, r_H H/c\ .$$

Combining with (3) gives

$$V_H=(2cr_HH)^{1/2}. \quad (4)$$

which applies to a given test particle at radius r_H. Another test particle at a different radius would have a different speed in accordance with $V_H \sim r_H^{1/2}$. This would be contrary to observation in the Deep MOND of spiral galaxies, and (4) accordingly cannot represent spiral galaxy flattening.

Our next objective is the time-dilation and corresponding Lorentzian test particle speed provided by the Schwarzschild solution of the Einstein equations. We have two choices on how to proceed:

(i) Isotropic photon speed in expanding space, or

(ii) Inwardly infinite photon speed in non-expanding space

where both give the same time dilation. More to the point, in choice (i) Einstein and Straus (1945) proved that space expansion for stipulated c=constant has no influence on the gravity field about an individual star, and in choice (ii) Rizzi et al. (2008) and Chamberlain (2015) showed that no predictable empirical effect or measurable interaction attends anisotropic light speed within nonexpanding space: the Schwarzschild solution

$$dt'/dt = (1-2GM/r_Sc^2)^{1/2} \quad (5)$$

applies in either case.

Assuming small M/r_S in (5) and expanding allows $dt'/dt = 1 - (G/c^2)M/r_S + \cdots$, which becomes the time-difference derivative

$$d\Delta t'/dt = -(GM/r_Sc^2) + \cdots \quad (6)$$

As before we combine $d\Delta t'/dt$ with (3), but this time yielding

$$V_S = (2GM/r_S)^{1/2} \quad (7)$$

for test particle speed. This result also fails to give radial invariance in Deep MOND in that V_S decreases with increasing radius.

Sub-Field Time Dilation. We now make the second "inductive jump", this to give the invariant time dilation around a massive entity centered at r=0—where the first "inductive jump", as earlier introduced, consisted of inwardly-infinite light speed combined with Hubble space-expansion to give $d\Delta t'/dt = -r_HH/c$. The (second) "inductive jump" is to combine (4) and (7) in the product $(V_S/c)(V_H/c)$ which, when "reversed" by the (linearized) Lorentz transform, yields the uniquely new time-dilation:

$$\begin{aligned} d\Delta t'/dt &= -\tfrac{1}{2}(V/c)^2 \\ &= -\tfrac{1}{2}V_SV_H/c^2 \\ &= -([GM/r_S][r_HcH])^{1/2}/c^2 \\ &= -(GMcH)^{1/2}/c^2 \end{aligned} \quad (8)$$

where the radii cancel. The Hubble parameter is defined radius-invariant (e.g., in our present epoch), and the sub-field time dilation is similarly invariant. Postulating steady, non-accelerating Hubble expansion however—i.e., $r_0H_0=rH$—yields a radius dependent time-dilation which, as shown immediately below, gives sub-field gravitational acceleration upon radial differentiation.

Sub-Field Gravitation. Sub-field time dilation (8) differs from Schwarzschild time dilation (6) by its independence of radial distance from the gravitational source in the given epoch. Despite its flat distribution, radial differentiation yields the corresponding gravitational acceleration. More to the point, while Schwarzschild gravitation emerges from radial differentiation of a non-uniform time-dilation, that is,

$$\begin{aligned} a &= -c^2\mathrm{d}(\mathrm{d}\Delta t'/\mathrm{d}t)/\mathrm{d}r_\mathrm{S} \\ &= \ \mathrm{d}(GM/r_\mathrm{S})/\mathrm{d}r_\mathrm{S} \\ &= -\, GM/r_\mathrm{S}{}^2 \end{aligned}$$

in the weak-field/non-relativistic limit, sub-field gravitation emerges from radial differentiation of a uniform time-dilation, i.e.,

$$\begin{aligned} a &= -c^2\mathrm{d}(\mathrm{d}\Delta t'/\mathrm{d}t)/\mathrm{d}r \\ &= \mathrm{d}((GMcH)^{1/2})/\mathrm{d}r \\ &= \mathrm{d}((GMcr_0H_0/r_\mathrm{H})^{1/2})/\mathrm{d}r \\ &= -\, ½\, (GMcH_0)^{1/2}/r \end{aligned}$$

where non-accelerating Hubble expansion, $rH{=}r_0H_0$, is postulated, as earlier noted, to (inductively) model empirical Deep MOND. This result, in turn, yields the relationship $a_0{=}cH_0/4$, upon comparison of "$a = -½(GMcH_0)^{1/2}/r = -(GMcH_0/4)^{1/2}/r$" with Milgrom's "$a = -(GMa_0)^{1/2}/r$".

Having introduced sub-field gravitation as the radial derivative of sub-field time dilation, the task now is to accommodate this development as a revision of general relativity. Our procedure in the present work is to revise the Schwarzschild solution rather than directly address the Einstein equations.

3.0 Schwarzschild Solution—Accommodation of Hubble Space-Expansion

3.1 Sub-Field Metric

The foregoing development addressed star gravitation in the far-field where acceleration predicted by standard theory has effectively vanished, leaving only the sub-field acceleration $a{=}-½(GMcH)^{1/2}/r$. In order to also accommodate general relativity thereby covering the near-to-far range it is necessary to postulate a metric for the sub-field and then employ the Einstein Equations for the complete solution.

In formulating the sub-field metric two conditions must be satisfied:

(a) Sub-field time dilation within any given epoch must satisfy $\mathrm{d}\Delta t'/\mathrm{d}t = -\,(GMcH)^{1/2}/c^2$; and

(b) Sub-field gravity must satisfy $a = \mathrm{d}(GMcH)^{1/2}/\mathrm{d}r = -\,½\,(GMcH)^{1/2}/r$.

Both requirements are satisfied by the metric

$$\mathrm{d}s^2 = -\,(1-(GMcH)^{1/2}/c^2)^2\, c^2\mathrm{d}t^2 + (1-([GMcH])^{1/2}/c^2)^{-2}\mathrm{d}r^2 + r^2\mathrm{d}\theta^{\,2} + r^2\sin^2\theta\mathrm{d}\phi^2 \qquad (9)$$

where the scale factor is dropped in the present study in recognition of its negligible variation during a photon transit across a planetary or galactic system.

A question at this point is whether (9) satisfies Einstein's field equations. Bypassing the details this may be demonstrated by direct substitution into the EFEs without the stress-energy tensor and the cosmological constant, i.e.,

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = 0$$

resulting in the corresponding (four) $E_{\mu\nu}$ relations. One can then solve

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = E_{\mu\nu}$$

—that is, follow the Schwarzschild procedure with $E_{\mu\nu}$ in place of $(8\pi G/c^4)T_{\mu\nu}$ —thereby recovering the metric and confirming the functional inter-relationship.[3]

3.2 Complete Solution

We now seek a revised Schwarzschild solution of the Einstein equations,

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = (8\pi G/c^4)T_{\mu\eta}\ , \qquad (10)$$

accounting for space expansion by way of the metric (9). With reference to Vojinovic's exposition (2010) of Schwarzschild's original solution, which we will follow, the first step is to rewrite (9) as:

$$\mathrm{d}\underline{s}^2 = -\, c^2\mathrm{d}\underline{t}^2 + \mathrm{d}\underline{r}^2 + \underline{r}^2\mathrm{d}\theta^2 + \underline{r}^2\sin^2\theta\,\mathrm{d}\phi^2$$

where time t and radius r have been respectively normalized by $\exp(2f(r))=(1-(GMcH)^{1/2}/c^2)^2$ and $\exp(2h(r))=(1-(GMcH)^{1/2}/c^2)^{-2}$.

The generalized metric that must be dominated by the Schwarzschild formulation in the "near field" while asymptotically approaching the sub-field formulation in the far-field may now be formed:

$$\mathrm{d}\underline{s}^2 = -\exp(2\underline{\underline{F}}(\underline{r}))\, c^2\mathrm{d}\underline{t}^2 + \exp(2\underline{\underline{H}}(\underline{r}))\,\mathrm{d}\underline{r}^2 + \underline{r}^2\mathrm{d}\theta^2 + \underline{r}^2\sin^2\theta\,\mathrm{d}\phi^2 \qquad (11)$$

where $\underline{\underline{F}}(\underline{r})$ and $\underline{\underline{H}}(\underline{r})$ are to be determined.

Equation (11) is the same metric addressed by Schwarzschild (and Vojinovic), and must yield the same Einstein tensor for the left-hand-side of (10) (primes denote derivatives):

$$\underline{G_{tt}} = -\,\underline{r}^{-2}\exp(2\underline{\underline{F}} - 2\underline{\underline{H}})\,(1 - 2\underline{r}\underline{\underline{H}}' - \exp(2\underline{\underline{H}}))\ , \qquad \underline{G_{rr}} = -\,\underline{r}^{-2}\,(1 + 2\underline{r}\underline{\underline{F}}' - \exp(2\underline{\underline{H}}))\ ,$$

$$\underline{G_{\theta\theta}} = -\,\underline{r}^{2}\exp(-2\underline{\underline{H}})\,[\underline{\underline{F}}'' + (\underline{\underline{F}}' + 1/\underline{r})(\underline{\underline{F}}' - \underline{\underline{H}}')]\ , \qquad \underline{G_{\phi\phi}} = G_{\theta\theta}\sin^2\theta$$

For the right-hand-side we adopt Vojinovic's "simplest possible stress-energy tensor, namely one that represents a static ball of radius R and density $\rho(r)$."[4] Within a stationary star only the diagonal components of the tensor are finite:

$$\underline{T_{tt}} = \underline{\rho} c^2\exp(2\underline{\underline{F}})\ , \quad \underline{T_{rr}} = \underline{p}\exp(2\underline{\underline{H}})\ , \quad \underline{T_{\theta\theta}} = \underline{p}\underline{r}^2\ , \quad \underline{T_{\phi\phi}} = \underline{p}\underline{r}^2\sin^2\theta\,.$$

---

[3] Alternatively, as demonstrated in the following section the Minkowski metric, comprising the fundamental solution, can be recovered by normalizing $\mathrm{d}t^2$ and $\mathrm{d}r^2$ by their respective coefficients.
[4] "Star" rather than "ball" will be employed in the present work.

and outside the star the terms are zero due to zero density and pressure.

Substituting the Einstein and stress-energy tensors into the field equations now gives three subsidiary differential equations:

- $\underline{t}$-$\underline{t}$ equation:

$$\underline{r}^{-2}\exp(2\underline{F})\mathrm{d}[\underline{r}\,(1-\exp(-2\underline{H}))]/\mathrm{d}\underline{r} = (8\pi G/c^2)\exp(2\underline{F})\,\underline{\rho}(\underline{r})\ ,$$

- $\underline{r}$-$\underline{r}$ equation:

$$\underline{r}^{-2}\,(1+2\underline{r}\underline{F}' - \exp(2\underline{H})) = (8\pi G/c^4)\exp(2\underline{H})\,\underline{p}(\underline{r})\ ,$$

- θ-θ and ϕ-ϕ equations (identical):

$$\underline{r}^2\exp(-2\underline{H})\,[\underline{F}''+(\underline{F}'+1/\underline{r})(\underline{F}'-\underline{H}')] = (8\pi G/c^4)\underline{r}^2\underline{p}(\underline{r}).$$

Solving for $\underline{F}$ and $\underline{H}$ gives

$$\underline{F}(\underline{r}) = ½\ln(1-2GM/\underline{r}c^2)$$

and

$$\underline{H}(\underline{r}) = -½\ln(1-2GM/\underline{r}c^2)$$

with M representing the star mass. Equation (11) accordingly becomes the original Schwarzschild solution irrespective of space expansion:

$$\mathrm{d}\underline{s}^2 = -(1-2GM/\underline{r}c^2)\,c^2\mathrm{d}\underline{t}^2 + (1-2GM/\underline{r}c^2)^{-1}\,\mathrm{d}\underline{r}^2 + \underline{r}^2\mathrm{d}\theta^2 + \underline{r}^2\sin^2\theta\mathrm{d}\phi^2\ .$$

Reversing the earlier normalization of t and r gives the metric solution external to the individual star gravitational field accounting for Hubble expansion:

$$\begin{aligned}\mathrm{d}s^2 &= -(1-2GM/\kappa rc^2)\,c^2\mathrm{d}t^2\,(1-(GMcH)^{1/2}/c^2)^2 \\ &\quad + (1-2GM/\kappa rc^2)^{-1}\mathrm{d}r^2(1-(GMcH)^{1/2}/c^2)^{-2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2 \\ &= -(1-2GM/rc^2)\,c^2\mathrm{d}t^2\,(1-(GMcH)^{1/2}/c^2)^2 \\ &\quad + (1-2GM/rc^2)^{-1}\mathrm{d}r^2(1-(GMcH)^{1/2}/c^2)^{-2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2 \qquad (12)\end{aligned}$$

where the higher order terms of $\kappa = (1-(GMcH)^{1/2}/c^2)$ may be ignored as negligible in the present study.

For zero space expansion (H=0) Schwarzschild's original solution is recovered (complying with the Correspondence Principle). However, for stipulated non-accelerating space expansion the Hubble parameter becomes radius-dependent thereby yielding the new, near-vanishing gravitational field with measurable effects around stars and galaxies, as addressed below.

4.0 Comparison with Earlier Theory

General relativity is known to be highly accurate within our Solar System, and it is appropriate to begin assessments of the present theory in this "nearby" domain. In this assessment we are primarily interested in the offset or adjustment of gravity itself and only secondarily in the several Einstein effects—e.g., Mercury orbit precession—in the close vicinity of the Sun. Later in this section we will, however, briefly address the Einstein effects in terms of the gravity-offset magnitudes near the Sun.

Because relativistic departures from Newtonian theory are near-vanishingly small throughout the planetary system we may significantly simplify by considering the weak field limit with negligible v/c.

4.1 Nonrelativistic, Weak-Field Limit

Linearizing (12) in M to satisfy the weak field requirement and restricting test particle speeds to the non-relativistic limit yields:

$$ds^2 = -(1-2GM/rc^2)\, c^2 dt^2 (1-(GMcH)^{1/2}/c^2)^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Transforming back to Cartesian space-coordinates gives

$$ds^2 = -(1-2MG/rc^2)\, c^2 dt^2 (1-(GMcH)^{1/2}/c^2)^2 + dx^2 + dy^2 + dz^2$$

where we see that "time is curved while space is flat". It is also seen, however—as the crucial consideration here—that time is additionally curved by the factor "$(1-(GMcH)^{1/2}/c^2)$" for stipulated non-accelerating Hubble expansion.

The differential equations for the geodesic line

$$d^2x^\lambda/d\tau^2 + \Gamma^\lambda{}_{\mu\nu}\,(dx^\mu/d\tau)(dx^\nu/d\tau) = 0$$

are employed to yield the four equations of motion:

$$d^2t/d\tau^2 = -\,\Gamma^t{}_{\mu t}\,(dx^\mu/d\tau)(dt/d\tau)$$

$$d^2x/d\tau^2 = -\,\Gamma^x{}_{tt}\,(dt/d\tau)^2$$

$$d^2y/d\tau^2 = -\,\Gamma^y{}_{tt}\,(dt/d\tau)^2$$

$$d^2z/d\tau^2 = -\,\Gamma^z{}_{tt}\,(dt/d\tau)^2$$

Evaluating the four Christoffel coefficients gives:

$$\begin{aligned} t:\quad d^2t/d\tau^2 = -\Big[&2\,(GMx/r^3c^2 + (GMcH)^{1/2}\,x/r^2c^2)\,dx/d\tau \\ &+ 2\,(GMy/r^3c^2 + (GMcH)^{1/2}\,y/r^2c^2)\,dy/d\tau \\ &+ 2\,(GMz/r^3c^2 + (GMcH)^{1/2}\,z/r^2c^2)\,dz/d\tau\Big](dt/d\tau) \end{aligned} \qquad (13)$$

$$x:\quad d^2x/d\tau^2 + (GMx/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}\,x/r^2)(dt/d\tau)^2 = 0$$

$$y:\quad d^2y/d\tau^2 + (GMy/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}\,y/r^2)(dt/d\tau)^2 = 0$$

$$z:\quad d^2z/d\tau^2 + (GMz/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}\,z/r^2)(dt/d\tau)^2 = 0$$

where, for example,

$$\Gamma^x{}_{tt} = \tfrac{1}{2}\, g^{x\sigma}\,(\partial_t g_{t\sigma} + \partial_t g_{t\sigma} - \partial_\sigma g_{tt})$$

with

$$[g_{\mu\nu}] = \begin{bmatrix} -(1-2MG/rc^2)\bullet(1-(GMcH)^{1/2}/c^2)^2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$[g^{\mu\nu}] = \begin{bmatrix} -\{(1-2MG/rc^2)\bullet(1-(GMcH)^{1/2}/c^2)^2\}^{-1} & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

giving

$$\Gamma^{x}{}_{tt} = (GMx/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}x/r^2).$$

Returning to the equation set (13), in the non-relativistic limit $(13)_1$ is $\mathrm{d}^2t/\mathrm{d}\tau^2 = 0$. Integrating and requiring τ be compatible with the affine structure of the problem (specifically, that τ and t have the same origin and scale) gives $\tau = t$. Equations $(13)_{2\text{-}4}$ become

$$\begin{aligned} x:&\quad \mathrm{d}^2x/\mathrm{d}t^2 + (GMx/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}x/r^2) = 0 \\ y:&\quad \mathrm{d}^2y/\mathrm{d}t^2 + (GMy/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}y/r^2) = 0 \\ z:&\quad \mathrm{d}^2z/\mathrm{d}t^2 + (GMz/r^3 + \tfrac{1}{2}\,(GMcH)^{1/2}\,z/r^2) = 0 \end{aligned} \tag{14}$$

For H finite (expanding universe), predicted gravitational acceleration is seen to exceed Newtonian gravitation and general relativity at all radii external to the star.

4.2 Influence of Sub-Field Gravitation on the Einstein Effects

Dividing the Sun's sub-field gravity by Newtonian gravity gives the relation $g_{SF}/g_N=[\tfrac{1}{2}(GMcH_0)^{1/2}/r]/[GM/r^2]=\tfrac{1}{2}(cH_0/GM)^{1/2}r$. Using this expression, we may compute the distance from the Sun where $g_{SF}/g_N = 1$, i.e., $r=2(GM/cH_0)^{1/2}\approx 4.4\text{E}11$ km or ~7000AU. Beyond this distance sub-field gravity becomes increasingly dominant—and for this we have empirical evidence as will be shown in the next section. For now we are interested in the degree to which sub-field gravitation affects or modifies space-time function from the surface of the Sun through the inner planets.

We may first observe that gravitation theory is fundamentally germane to determining the Solar mass, and if Newtonian gravitation is "off" by some margin, however small, the calculated Solar mass will be similarly affected. This is of course a complication in assessing the influence of sub-field gravitation on planetary orbits as well as on the several Einstein effects. Nevertheless, we can compare sub-field gravity against Newtonian gravity over the limited range near the Sun and so achieve some insight.

Recalling the earlier expression $g_{SF}/g_N = ½\,(cH_0/GM)^{1/2}r$ and noting $g_{SF}/g_N = 1$ when r=6,588 AU, we may immediately determine $g_{SF}/g_N = 6{,}588^{-1}$ at earth orbit. Moving closer to the Sun we obtain $g_{SF}/g_N = 9{,}883^{-1}$ for Venus and $14{,}821^{-1}$ for Mercury. These marginal departures from Newtonian gravity will be further reduced when the Solar and planetary masses are adjusted to minimize the ephemerides.

Turning now to the Einstein effects, because space-time curvature is fundamentally germane to the several gravitational effects—i.e., Mercury orbit precession, time dilation, radiation redshift, and light deflection—and sub-field gravity is a small/marginal adjustment of local space-time curvature, it follows that we may expect a similarly small modification or adjustment of the magnitudes of Einstein's predictions. As an example, the measured 574" per century precession of Mercury is increased by ~0.1" in the present theory, and Einstein's 43" per century precession is accordingly increased to ~43.1" per century. However, rigorous comparisons of predictions and measurements across the Solar System are of course necessary, and these studies will help direct deeper modifications of gravitational theory.

5.0 Theory versus Measurement

Comparison of present theory with measurements of star dynamics is presented for two cases: (a) wide binary stars with centrifugal accelerations at and near Milgrom's universal gravitational constant; and (b) spiral galaxies.

5.1 Wide Binary-Star Rotation Flattening

Figure 1 shows measured and predicted wide-binary relative velocity versus separations—in the particular "gravity domain" where the present sub-field gravity and orthodox gravity (Newtonian and general relativistic) are about the same order. Crosses give the Hipparcos and SDSS data with horizontal and vertical bars representing the uncertainties in separation and relative velocity. Considerable uncertainty is evident due to several effects, including slowly rotating stars (~1 km/sec) with incompletely known orbital parameters (e.g., orbit inclination and eccentricity) and a signal-to-noise ratio of about 1.7. Positions of Milgrom's universal acceleration parameter a_0 and the closely associated cH_0 are given by the vertical dashed lines. (Recall, however, that the more meaningful comparison is between a_0 and $cH_0/4$, and so the two lines would be nearly coincident.)

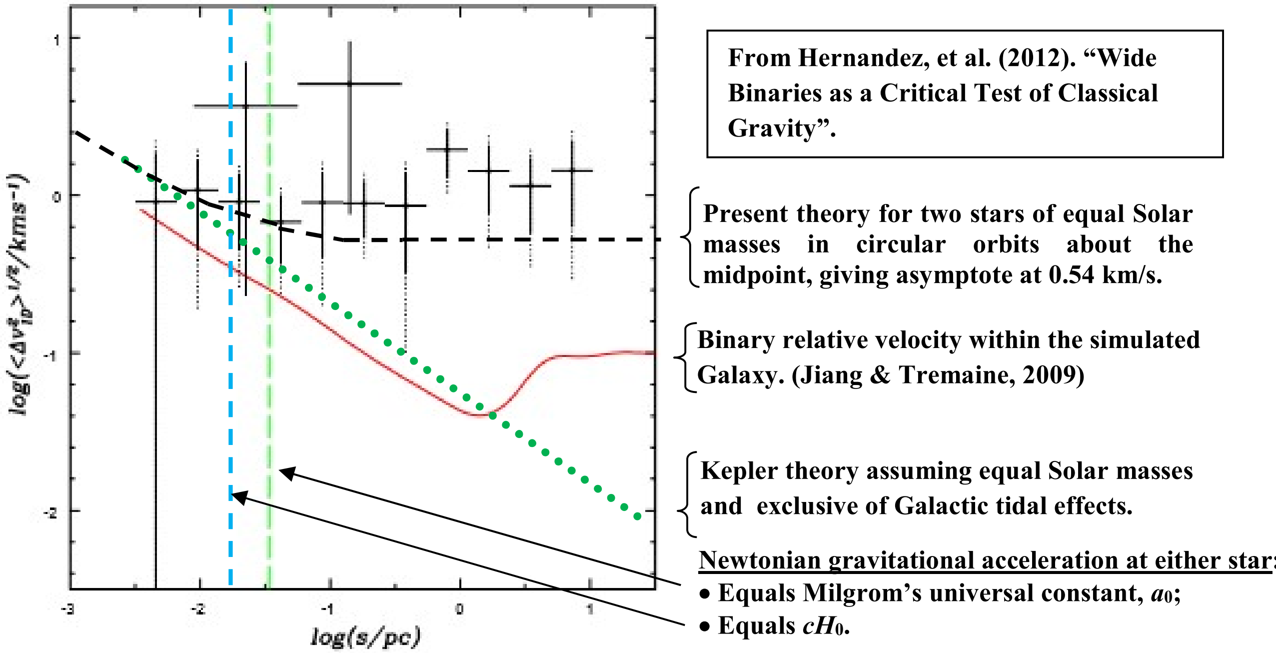

Figure 1. Approximate agreement exists between present theory (dashed curve) and the combined Hipparcos and SDSS samples of wide binaries in the near-field to far-field transition, while Newtonian Galactic dynamics and Kepler predictions fall short beyond log(s/pc) ≈ −1.5. (Adapted from Hernandez et al. (2012).)

Also shown in the figure are three theoretical curves. The two falling well below the data are standard theory predictions assuming equal Solar masses, where Kepler's prediction irrespective of Galactic tides is the (dotted) straight line and the corresponding (solid line) prediction accounting for tidal effects (Jiang and Tremaine (2010)) tracks approximately 35% below the Kepler line until approaching the Jacobi radius at 1.7 parsecs. As the third of the three theories (dashed line), the present revision of the Schwarzschild solution "lifts off" the Kepler line at about 0.01 pc, in agreement with the measured onset of rotation flattening, and then passes through the lower range of measured relative velocities before leveling off at ΔV=0.54 km/sec.

5.2 Spiral Galaxy Rotation Flattening

If we assume that measured spiral galaxy rotation in the asymptotic limit is sufficiently far-field to allow mass concentration effectively at the center, then (14) permits the relation $V_f=(GMcH_0/4)^{1/4}$ (i.e., using $V_f^2/r=(GMcH_0/4)^{1/2}/r$) for comparison with the Tully-Fisher relation. The comparison is shown in Figure 3 in terms of outer rotation velocity V_f (varying from ~10 km/s to ~300 km/s) versus total galaxy mass M_b (~10^7 to ~10^{12} Solar masses), where Milgrom's Deep MOND correlation (dotted line) is nearly coincident with the present theory.

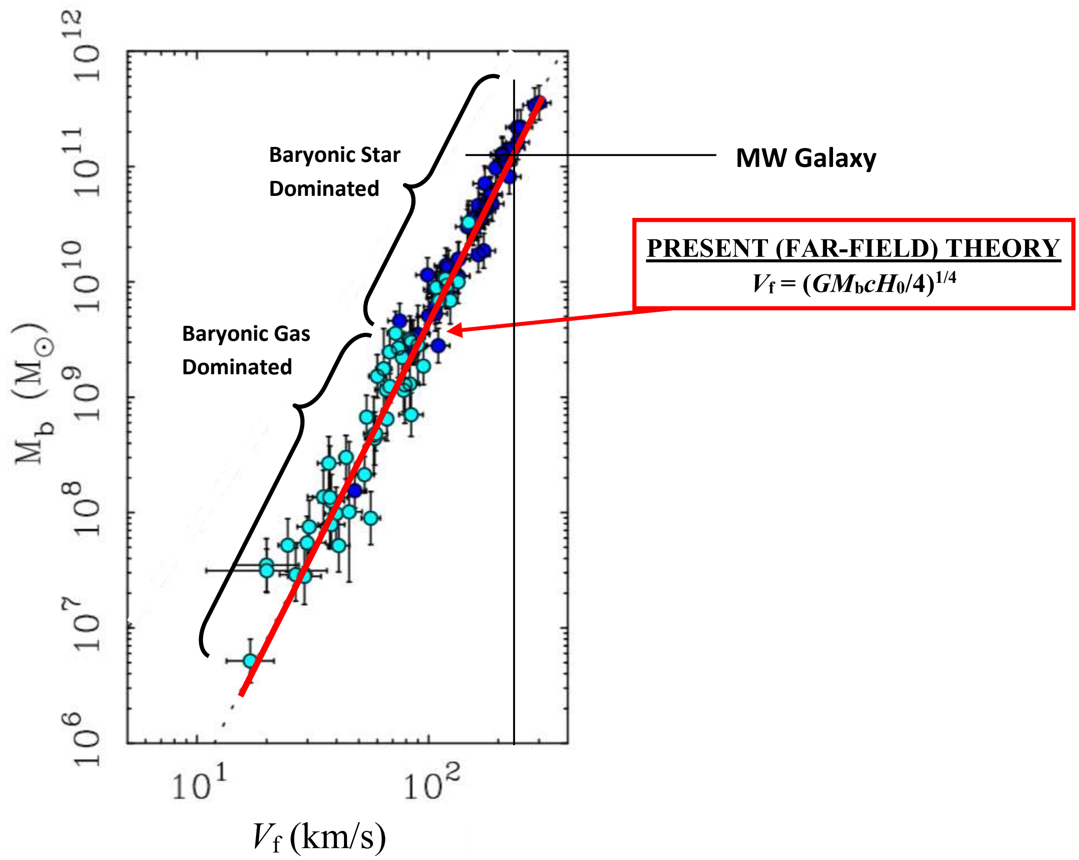

Figure 2: Present far-field theory compared with the Tully-Fisher relation for galaxies with well-measured outer velocities V_f. Milgrom's empirical Deep MOND is effectively "eclipsed" by the red-line theory. (From Famaey and McGaugh (2012)).

The present model (solid line) is seen to only marginally depart from Milgrom's empirical Deep MOND (dotted line), where both the empirical fit and theoretical prediction closely match the measurements across five orders of galactic mass M_b. (Milky Way galaxy is at the upper right.)

6.0 Conclusion

Inwardly infinite photon speed (with $c/2$ outward) in place of Einstein's isotropic c=constant light speed is the essential departure of the present theory from orthodox or standard relativity physics. Were it not for Hubble expansion the departure would be without new predictive or explanatory capability, and therefore without scientific importance. But when combined with Hubble space-expansion, inwardly infinite photon speed gives radially increasing time dilation $d\Delta t'/dt = -r_H H/c$ and this combined with the radially decreasing Schwarzschild time dilation $d\Delta t'/dt = -(GM/r_S c^2)$ yields a new time dilation $d\Delta t'/dt = -(GMcH)^{1/2}/c^2$ that is independent of radial distance in any given epoch. Radial differentiation for steady, non-accelerating Hubble space-expansion then yields the $1/r$ gravitational acceleration necessary to explain spiral galaxy rotation flattening without recourse to dark matter. These results contradict the Einstein and Straus (1945) "no influence" conclusion, but their analysis was based on the c=constant *special case* rather than the more fundamental anisotropic light speed.

Mordehai Milgrom's empirical Deep MOND for correlating far-field galactic dynamics has been highly advantageous in arriving at the underlying relativistic theory. While Deep MOND did not permit an analytic determination in the sense of a derivation, it greatly facilitated inductive investigation leading to the new theory. This specific effort began soon after the presentation of "Fundamentally Anisotropic Light-Velocity at the Foundation of Classical Physics" during the AAAS-PD annual conference in San Francisco (2015; title changed) and shortly revealed a relativistic relationship between Einstein's same-motion acceleration/synchrony (1907) and Hubble space expansion in the limit $\beta = v/c \rightarrow 1$ (for which inward photon speed is infinite); more to the point, cH_0 emerged having approximately the same order-of-magnitude as Milgrom's universal acceleration parameter a_0—i.e., 6.3E-13 km/s^2 versus Milgrom's 1.2E-13 km/s^2 (the identity $a_0=cH_0/4$ was recognized later in the paper). Although not directly useful this "clue" additionally aided the theory development. Definition of the corresponding metric was followed by integration within the Schwarzschild geometry and problem definition to give the (additive) solution throughout the spatial domain external to the star.

Because Milgrom's Deep MOND guided inductive development of the theory, we may expect, and have found, corresponding agreement between calculated and measured far-field rotation of spiral galaxies. More towards the opposite extreme where Newtonian mechanics dominates we found that the additive Schwarzschild plus sub-field solution gave a smaller than 10^{-4} increase of gravitation at Mercury orbit. Calculated crossover between $1/r^2$ Newtonian gravitation and $1/r$ sub-field gravity occurred at ~7000 AU from the Sun, in approximate agreement with wide-binary data (after assigning Solar masses to the orbiting stars).

The revised Schwarzschild geometry preserves the several Einstein effects in the Solar vicinity as a consequence of the near vanishing increase of gravitational acceleration at and inside Mercury orbit. An assessment of ephemerides residuals throughout the Solar System, after accounting for the present adjustment of standard gravitation theory, would critique the present theory, and illuminate the path to deeper theory.

REFERENCES

1. Einstein, A. and E. Straus (1945). The Influence of the Expansion of Space on the Gravitational Fields Surrounding the Individual Stars. *Reviews of Modern Physics*, Vol. 17, Numbers 2 and 3.
2. Rizzi, G., M.L. Ruggiero, and A. Serafini (2008). Synchronization Gauges and the Principles of Special Relativity. *Found. Phys.* 34 1885.
3. T. E. Chamberlain. (2015). Fundamentally Anisotropic Light-Velocity at the Foundation of Classical Physics (title changed). Presented at the *96th Annual AAAS-PD Meeting*, 14-17 June 2015, San Francisco, California.
4. Milgrom, M. (1983). A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis. *Astrophys. J.*, 270, 365–370.
5. Einstein, A. [1907] (1977). Einstein's Comprehensive 1907 Essay on Relativity. Part III. *Am. J. Phys*., Vol. 45, Number 10.
6. Vojinovic, M. (2010). Schwarzschild Solution in General Relativity. *Seminar on Schwarzschild Solution of Einstein Equations*, Given at *GFM*, *Complexo Interdisciplinar*, Lisbon.
7. Jiang, Y. F., and S. Tremaine (2010). MNRAS, 401, 977.
8. Hernandez, X., M.A. Jimenez, and C. Allen (2012). Wide Binaries as a Critical Test of Classical Gravity. arXiv:1105.1873 v4.
9. Famaey, B. and S. McGaugh (2012). Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions. *Living Rev. Relativity*, 15.